

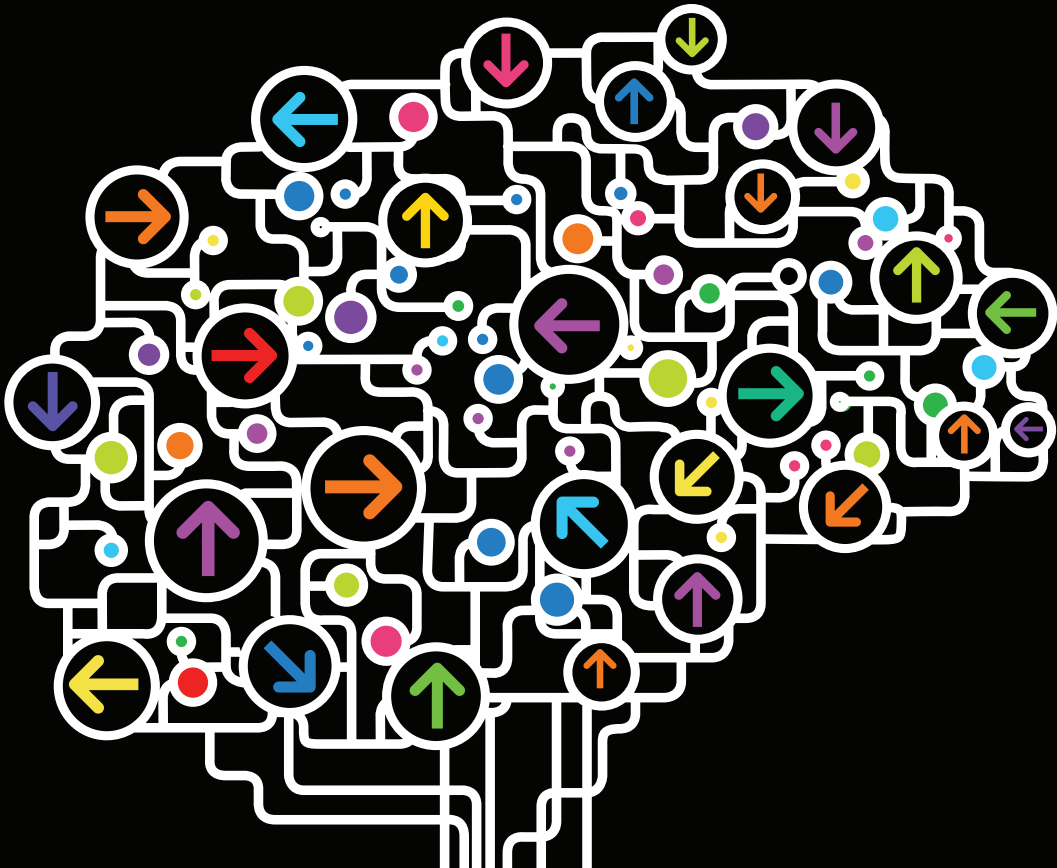
GLOBAL
EDITION



Feedback Control of Dynamic Systems

SEVENTH EDITION

Gene F. Franklin • J. David Powell • Abbas Emami-Naeini



ALWAYS LEARNING

PEARSON

Feedback Control of Dynamic Systems

Seventh Edition

Global Edition

Gene F. Franklin

Stanford University

J. David Powell

Stanford University

Abbas Emami-Naeini

SC Solutions, Inc.

Global Edition contributions by

Sanjay H.S.

M.S. Ramaiah College of Engineering

PEARSON

Boston Columbus Indianapolis New York San Francisco Upper Saddle River
Amsterdam Cape Town Dubai London Madrid Milan Munich Paris Montreal Toronto
Delhi Mexico City São Paulo Sydney Hong Kong Seoul Singapore Taipei Tokyo

Vice President and Editorial Director,
ECS: *Marcia J. Horton*
Executive Editor: *Holly Stark*
Editorial Assistant: *Sandra Rodriguez*
Executive Marketing Manager: *Tim Galligan*
Marketing Assistant: *Jon Bryant*
Senior Managing Editor: *Scott Disanno*
Production Program Manager: *Clare Romeo*
Production Project Manager: *Rose Kernan*
Head, Learning Asset Acquisition, Global Edition: *Laura Dent*
Acquisitions Editor, Global Edition: *Karthik Subramaniam*

Project Editor, Global Edition: *Anuprova Dey Chowdhuri*
Director of Operations: *Nick Sklitsis*
Operations Specialist: *Linda Sager*
Cover Designer: *Lumina Datamatics Ltd.*
Permissions Project Manager: *Rachel Youdelman*
Full-service Project Management: *Pavithra Jayapaul*
Composition: *Jouve India*
Printer/Binder: *Courier Westford*
Cover Printer: *Courier Westford*
Cover Photograph: *Shutterstock*
Typeface: *10/12 Times*

Pearson Education Limited
Edinburgh Gate
Harlow
Essex CM20 2JE
England

and Associated Companies throughout the world

Visit us on the World Wide Web at:
www.pearsonglobaleditions.com

© Pearson Education Limited 2015

The rights of John Lewis and William Loftus to be identified as the authors of this work have been asserted by them in accordance with the Copyright, Designs and Patents Act 1988.

Authorized adaptation from the United States edition, entitled Feedback Control of Dynamic System, 7th edition, ISBN 978-0-13-349659-8, by Gene F. Franklin, J. David Powell and Abbas Emami-Naeini, published by Pearson Education © 2015.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without either the prior written permission of the publisher or a license permitting restricted copying in the United Kingdom issued by the Copyright Licensing Agency Ltd, Saffron House, 6–10 Kirby Street, London EC1N 8TS.

All trademarks used herein are the property of their respective owners. The use of any trademark in this text does not vest in the author or publisher any trademark ownership rights in such trademarks, nor does the use of such trademarks imply any affiliation with or endorsement of this book by such owners.

Microsoft and/or its respective suppliers make no representations about the suitability of the information contained in the documents and related graphics published as part of the services for any purpose. All such documents and related graphics are provided “as is” without warranty of any kind. Microsoft and/or its respective suppliers hereby disclaim all warranties and conditions with regard to this information, including all warranties and conditions of merchantability, whether express, implied or statutory, fitness for a particular purpose, title and non-infringement. In no event shall Microsoft and/or its respective suppliers be liable for any special, indirect or consequential damages or any damages whatsoever resulting from loss of use, data or profits, whether in an action of contract, negligence or other tortious action, arising out of or in connection with the use or performance of information available from the services.

The documents and related graphics contained herein could include technical inaccuracies or typographical errors. Changes are periodically added to the information herein. Microsoft and/or its respective suppliers may make improvements and/or changes in the product(s) and/or the program(s) described herein at any time. Partial screen shots may be viewed in full within the software version specified.

Microsoft® and Windows® are registered trademarks of the Microsoft Corporation® in the U.S.A. and other countries. This book is not sponsored or endorsed by or affiliated with the Microsoft Corporation.

ISBN 10: 1-29-206890-6
ISBN 13: 978-1-29-206890-9

10 9 8 7 6 5 4 3 2 1
14 13 12 11 10

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library

Printed and bound by Courier Westford

The publisher's policy is to use paper manufactured from sustainable forests.

To the memory of Gene F. Franklin

Contents

Preface 13

1 An Overview and Brief History of Feedback Control 21

A Perspective on Feedback Control	21
Chapter Overview	22
1.1 A Simple Feedback System	23
1.2 A First Analysis of Feedback	26
1.3 Feedback System Fundamentals	30
1.4 A Brief History	31
1.5 An Overview of the Book	37
Summary	39
Review Questions	39
Problems	40

2 Dynamic Models 43

A Perspective on Dynamic Models	43
Chapter Overview	44
2.1 Dynamics of Mechanical Systems	44
2.1.1 Translational Motion	44
2.1.2 Rotational Motion	51
2.1.3 Combined Rotation and Translation	59
2.1.4 Complex Mechanical Systems (W)**	62
2.1.5 Distributed Parameter Systems	62
2.1.6 Summary: Developing Equations of Motion for Rigid Bodies	64
2.2 Models of Electric Circuits	65
2.3 Models of Electromechanical Systems	70
2.3.1 Loudspeakers	70
2.3.2 Motors	72
2.3.3 Gears	76
△ 2.4 Heat and Fluid-Flow Models	77
2.4.1 Heat Flow	78
2.4.2 Incompressible Fluid Flow	81
2.5 Historical Perspective	88
Summary	91
Review Questions	91
Problems	92

**Sections with (W) indicates that additional material is located on the web at www.FPE7e.com.

3 Dynamic Response 104

A Perspective on System Response	104
Chapter Overview	105
3.1 Review of Laplace Transforms	105
3.1.1 Response by Convolution	106
3.1.2 Transfer Functions and Frequency Response	111
3.1.3 The \mathcal{L} -Laplace Transform	121
3.1.4 Properties of Laplace Transforms	123
3.1.5 Inverse Laplace Transform by Partial-Fraction Expansion	125
3.1.6 The Final Value Theorem	127
3.1.7 Using Laplace Transforms to Solve Differential Equations	129
3.1.8 Poles and Zeros	131
3.1.9 Linear System Analysis Using Matlab [®]	132
3.2 System Modeling Diagrams	138
3.2.1 The Block Diagram	138
3.2.2 Block-Diagram Reduction Using Matlab	142
3.2.3 Mason's Rule and the Signal Flow Graph (W)	143
3.3 Effect of Pole Locations	143
3.4 Time-Domain Specifications	151
3.4.1 Rise Time	152
3.4.2 Overshoot and Peak Time	152
3.4.3 Settling Time	154
3.5 Effects of Zeros and Additional Poles	157
3.6 Stability	166
3.6.1 Bounded Input–Bounded Output Stability	167
3.6.2 Stability of LTI Systems	168
3.6.3 Routh's Stability Criterion	169
△ 3.7 Obtaining Models from Experimental Data: System Identification (W)	176
△ 3.8 Amplitude and Time Scaling (W)	176
3.9 Historical Perspective	176
Summary	177
Review Questions	179
Problems	179

4 A First Analysis of Feedback 200

A Perspective on the Analysis of Feedback	200
Chapter Overview	201
4.1 The Basic Equations of Control	202
4.1.1 Stability	203
4.1.2 Tracking	204
4.1.3 Regulation	205
4.1.4 Sensitivity	206

4.2	Control of Steady-State Error to Polynomial Inputs: System Type	208
4.2.1	System Type for Tracking	209
4.2.2	System Type for Regulation and Disturbance Rejection	214
4.3	The Three-Term Controller: PID Control	216
4.3.1	Proportional Control (P)	216
4.3.2	Integral Control (I)	218
4.3.3	Derivative Control (D)	221
4.3.4	Proportional Plus Integral Control (PI)	221
4.3.5	PID Control	222
4.3.6	Ziegler–Nichols Tuning of the PID Controller	226
4.4	Feedforward Control by Plant Model Inversion	232
△	4.5 Introduction to Digital Control (W)	234
△	4.6 Sensitivity of Time Response to Parameter Change (W)	235
4.7	Historical Perspective	235
	Summary	237
	Review Questions	238
	Problems	238

5 The Root-Locus Design Method 254

	A Perspective on the Root-Locus Design Method	254
	Chapter Overview	255
5.1	Root Locus of a Basic Feedback System	255
5.2	Guidelines for Determining a Root Locus	260
5.2.1	Rules for Determining a Positive (180°) Root Locus	262
5.2.2	Summary of the Rules for Determining a Root Locus	268
5.2.3	Selecting the Parameter Value	269
5.3	Selected Illustrative Root Loci	271
5.4	Design Using Dynamic Compensation	284
	5.4.1 Design Using Lead Compensation	286
	5.4.2 Design Using Lag Compensation	290
	5.4.3 Design Using Notch Compensation	292
△	5.4.4 Analog and Digital Implementations (W)	294
5.5	A Design Example Using the Root Locus	295
5.6	Extensions of the Root-Locus Method	301
	5.6.1 Rules for Plotting a Negative (0°) Root Locus	301
△	5.6.2 Consideration of Two Parameters	304
△	5.6.3 Time Delay (W)	306
5.7	Historical Perspective	307

Summary	309
Review Questions	310
Problems	311

6 The Frequency-Response Design Method 328

A Perspective on the Frequency-Response Design Method	328
Chapter Overview	329
6.1 Frequency Response	329
6.1.1 Bode Plot Techniques	337
6.1.2 Steady-State Errors	350
6.2 Neutral Stability	351
6.3 The Nyquist Stability Criterion	353
6.3.1 The Argument Principle	354
6.3.2 Application of The Argument Principle to Control Design	355
6.4 Stability Margins	368
6.5 Bode's Gain-Phase Relationship	377
6.6 Closed-Loop Frequency Response	381
6.7 Compensation	383
6.7.1 PD Compensation	383
6.7.2 Lead Compensation (W)	384
6.7.3 PI Compensation	394
6.7.4 Lag Compensation	395
6.7.5 PID Compensation	401
6.7.6 Design Considerations	407
6.7.7 Specifications in Terms of the Sensitivity Function	409
6.7.8 Limitations on Design in Terms of the Sensitivity Function	414
6.8 Time Delay	418
6.8.1 Time Delay via the Nyquist Diagram (W)	420
6.9 Alternative Presentation of Data	420
6.9.1 Nichols Chart	420
6.9.2 The Inverse Nyquist Diagram (W)	424
6.10 Historical Perspective	424
Summary	425
Review Questions	428
Problems	428

7 State-Space Design 453

A Perspective on State-Space Design	453
Chapter Overview	454
7.1 Advantages of State-Space	454
7.2 System Description in State-Space	456
7.3 Block Diagrams and State-Space	462

7.4	Analysis of the State Equations	464
7.4.1	Block Diagrams and Canonical Forms	465
7.4.2	Dynamic Response from the State Equations	477
7.5	Control-Law Design for Full-State Feedback	483
7.5.1	Finding the Control Law	484
7.5.2	Introducing the Reference Input with Full-State Feedback	493
7.6	Selection of Pole Locations for Good Design	497
7.6.1	Dominant Second-Order Poles	497
7.6.2	Symmetric Root Locus (SRL)	499
7.6.3	Comments on the Methods	508
7.7	Estimator Design	509
7.7.1	Full-Order Estimators	509
7.7.2	Reduced-Order Estimators	515
7.7.3	Estimator Pole Selection	519
7.8	Compensator Design: Combined Control Law and Estimator (W)	521
7.9	Introduction of the Reference Input with the Estimator (W)	534
7.9.1	General Structure for the Reference Input	535
7.9.2	Selecting the Gain	544
7.10	Integral Control and Robust Tracking	545
7.10.1	Integral Control	546
△	7.10.2 Robust Tracking Control: The Error-Space Approach	548
△	7.10.3 Model-Following Design	559
△	7.10.4 The Extended Estimator	563
△	7.11 Loop Transfer Recovery	567
△	7.12 Direct Design with Rational Transfer Functions	572
△	7.13 Design for Systems with Pure Time Delay	576
7.14	Solution of State Equations (W)	579
7.15	Historical Perspective	579
	Summary	582
	Review Questions	585
	Problems	586

8

Digital Control 610

A Perspective on Digital Control	610
Chapter Overview	611
8.1 Digitization	611
8.2 Dynamic Analysis of Discrete Systems	614
8.2.1 z -Transform	614
8.2.2 z -Transform Inversion	615

	8.2.3	Relationship Between s and z	617
	8.2.4	Final Value Theorem	619
8.3		Design Using Discrete Equivalents	621
	8.3.1	Tustin's Method	622
	8.3.2	Zero-Order Hold (ZOH) Method	625
	8.3.3	Matched Pole–Zero (MPZ) Method	627
	8.3.4	Modified Matched Pole–Zero (MMPZ) Method	631
	8.3.5	Comparison of Digital Approximation Methods	632
	8.3.6	Applicability Limits of the Discrete Equivalent Design Method	633
8.4		Hardware Characteristics	633
	8.4.1	Analog-to-Digital (A/D) Converters	634
	8.4.2	Digital-to-Analog Converters	634
	8.4.3	Anti-Alias Prefilters	635
	8.4.4	The Computer	636
8.5		Sample-Rate Selection	637
	8.5.1	Tracking Effectiveness	638
	8.5.2	Disturbance Rejection	638
	8.5.3	Effect of Anti-Alias Prefilter	639
	8.5.4	Asynchronous Sampling	640
△ 8.6		Discrete Design	640
	8.6.1	Analysis Tools	641
	8.6.2	Feedback Properties	642
	8.6.3	Discrete Design Example	643
	8.6.4	Discrete Analysis of Designs	646
8.7		Discrete State-Space Design Methods (W)	648
8.8		Historical Perspective	648
		Summary	649
		Review Questions	651
		Problems	651

9 Nonlinear Systems 657

		A Perspective on Nonlinear Systems	657
		Chapter Overview	658
9.1		Introduction and Motivation: Why Study Nonlinear Systems?	659
9.2		Analysis by Linearization	661
	9.2.1	Linearization by Small-Signal Analysis	661
	9.2.2	Linearization by Nonlinear Feedback	666
	9.2.3	Linearization by Inverse Nonlinearity	667
9.3		Equivalent Gain Analysis Using the Root Locus	668
	9.3.1	Integrator Antiwindup	675

9.4	Equivalent Gain Analysis Using Frequency Response: Describing Functions	678
9.4.1	Stability Analysis Using Describing Functions	685
△ 9.5	Analysis and Design Based on Stability	690
9.5.1	The Phase Plane	690
9.5.2	Lyapunov Stability Analysis	697
9.5.3	The Circle Criterion	703
9.6	Historical Perspective	710
	Summary	711
	Review Questions	711
	Problems	712

10 Control System Design: Principles and Case Studies 723

	A Perspective on Design Principles	723
	Chapter Overview	724
10.1	An Outline of Control Systems Design	725
10.2	Design of a Satellite's Attitude Control	731
10.3	Lateral and Longitudinal Control of a Boeing	747 749
10.3.1	Yaw Damper	753
10.3.2	Altitude-Hold Autopilot	761
10.4	Control of the Fuel–Air Ratio in an Automotive Engine	767
10.5	Control of the Read/Write Head Assembly of a Hard Disk	775
10.6	Control of RTP Systems in Semiconductor Wafer Manufacturing	783
10.7	Chemotaxis or How <i>E. Coli</i> Swims Away from Trouble	797
10.8	Historical Perspective	806
	Summary	808
	Review Questions	810
	Problems	810

Appendix A Laplace Transforms 824

A.1	The \mathcal{L} -Laplace Transform	824
A.1.1	Properties of Laplace Transforms	825
A.1.2	Inverse Laplace Transform by Partial-Fraction Expansion	833
A.1.3	The Initial Value Theorem	836
A.1.4	Final Value Theorem	837

Appendix B Solutions to the Review Questions 839

Appendix C Matlab Commands 855

Bibliography 860

Index 868

List of Appendices on the web at www.fpe7e.com

Appendix WA: A Review of Complex Variables

Appendix WB: Summary of Matrix Theory

Appendix WC: Controllability and Observability

Appendix WD: Ackermann's Formula for Pole Placement

Appendix W2.1.4: Complex Mechanical Systems

Appendix W3.2.3: Mason's Rule and Signal Flow Graph

Appendix W3.6.3.1: Routh Special Cases

Appendix W3.7: System Identification

Appendix W3.8: Amplitude and Time Scaling

Appendix W4.1.4.1: The Filtered Case

Appendix W4.2.2.1: Truxal's Formula for the Error Constants

Appendix W4.5: Introduction to Digital Control

Appendix W4.6: Sensitivity of Time Response to Parameter Change

Appendix W5.4.4: Analog and Digital Implementations

Appendix W5.6.3: Root Locus with Time Delay

Appendix W6.7.2: Digital Implementation of Example 6.15

Appendix W6.8.1: Time Delay via the Nyquist Diagram

Appendix W6.9.2: The Inverse Nyquist Diagram

Appendix W7.8: Digital Implementation of Example 7.31

Appendix W7.9: Digital Implementation of Example 7.33

Appendix W7.14: Solution of State Equations

Appendix W8.7: Discrete State-Space Design Methods

Preface

In this Seventh Edition we again present a text in support of a first course in control and have retained the best features of our earlier editions. For this edition, we have responded to a survey of users by adding some material (for example, gears in Chapter 2) and moved other little-used material from the printed book (for example, digital control in the early chapters) to a website that is fully accessible to readers. We have also updated the text throughout so that it uses the improved features of Matlab[®]. But perhaps the biggest jolt to our readers is that we succumbed to the times, and changed the notation used for the state-space description from $(\mathbf{F}, \mathbf{G}, \mathbf{H}, \mathbf{J})$ to $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$! In past editions, our loyalty to the early pioneers in the state-space approach overwhelmed our ability to accept reality, and we stayed with the classical notation. However, for this edition, we decided the time has come to accept the reality that the dominant notation in the field today is $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$ as is the notation in Matlab. We have also added a section on feedforward control to Chapter 4 and a model-following section to Chapter 7. In addition, the presentation of PID control has been improved as has some of the Laplace transform material. We strive to equip control system designers with the theory, basic design methods, and an introduction to computer-aided design methods. At the same time, we also strive to equip designers with a basic understanding so that computer results can be guided and verified. The case studies in Chapter 10 have been retained and updated where needed. Finally, in order to guide the reader in finding specific topics, both in the text and on our website, we have expanded the table of contents to include entries for material that is on the website as well as in the printed book.

The basic structure of the book is unchanged and we continue to combine analysis with design using the three approaches of the root locus, frequency response, and state-variable equations. The text continues to include many carefully worked out examples to illustrate the material. As before, we provide a set of review questions at the end of each chapter with answers in the back of the book to assist the students in verifying that they have learned the material.

In the three central chapters on design methods we continue to expect the students to learn how to perform the very basic calculations by hand and make a rough sketch of a root locus or Bode plot as a sanity check on the computer results and as an aid to design. However, we introduce the use of Matlab early on in recognition of the universal use of software tools in control analysis and design. Furthermore, in recognition of the fact that very few instructors were using the early material on Digital Control in Chapters 4, 5, and 6 in the sixth edition, that material was moved to our website and Chapter 8 was modified so that it provides a stand-alone introduction to Digital Control. For those instructors wanting to include the digital implementation of controllers early in their teaching, the material can



www.FPE7e.com QR Code

be downloaded and used without change from the order that existed in the sixth edition or the students can be directed to the material in Chapter 8. As before, we have prepared a collection of all the Matlab files (both “m” files and Simulink[®] “mdl” files) used to produce the figures in the book. These are available along with the advanced material described above at our website at *www.FPE7e.com*

New to this Edition

We feel that this Seventh Edition presents the material with good pedagogical support, provides strong motivation for the study of control, and represents a solid foundation for meeting the educational challenges. We introduce the study of feedback control, both as a specialty in itself and as support for many other fields.

A more detailed list of the changes is:

- Added new section on Fundamentals to Chapter 1
- Added new section on Gears to Chapter 2
- Updated Matlab commands throughout the book in order to utilize current capabilities of the software
- Rewrote section on the Laplace transform and frequency response in Chapter 3
- Rewrote section on PID control in Chapter 4
- Added section on Feedforward control in Chapter 4
- Moved the section on digital control in Chapter 4 to a dedicated website for the book (www.FPE7e.com)
- Revised section in Chapter 4 on the effect of zeros on a system
- Moved the sections on digital control and time delay in Chapter 5 to the website
- Moved the sections on digital control in Chapter 6 to the website
- Rewrote sections on stability and compensation in Chapter 6 for clarity and consistency with current standards in the industry
- Expanded discussion of Nichols plots in Chapter 6
- Moved sections of digital control in Chapter 7 to the website
- Revised notation of the state-space system from **(F, G, H, J)** to **(A, B, C, D)** in Chapters 7, 9, and 10.
- To prevent any ambiguity, the notation for the compensation was changed from $D(s)$ to $D_c(s)$ throughout the text because of the change in the state-space notation

- Added the model-following procedure to Chapter 7
- Several sections were rewritten in Chapter 8 for clarity
- Added section on the ZOH approximate method in Chapter 8
- Updated the engine control example and substantially revised the system biology case study in Chapter 10
- Approximately 20% of the problems in the book are revised or new in all chapters

Addressing the Educational Challenges

Some of the educational challenges facing students of feedback control are long-standing; others have emerged in recent years. Some of the challenges remain for students across their entire engineering education; others are unique to this relatively sophisticated course. Whether they are old or new, general or particular, the educational challenges we perceived were critical to the evolution of this text. Here we will state several educational challenges and describe our approaches to each of them.

- **CHALLENGE** *Students must master design as well as analysis techniques.*

Design is central to all of engineering and especially so to control systems. Students find that design issues, with their corresponding opportunities to tackle practical applications, are particularly motivating. But students also find design problems difficult because design problem statements are usually poorly posed and lack unique solutions. Because of both its inherent importance and its motivational effect on students, design is emphasized throughout this text so that confidence in solving design problems is developed from the start.

The emphasis on design begins in Chapter 4 following the development of modeling and dynamic response. The basic idea of feedback is introduced first, showing its influence on disturbance rejection, tracking accuracy, and robustness to parameter changes. The design orientation continues with uniform treatments of the root locus, frequency response, and state variable feedback techniques. All the treatments are aimed at providing the knowledge necessary to find a good feedback control design with no more complex mathematical development than is essential to clear understanding.

Throughout the text, examples are used to compare and contrast the design techniques afforded by the different design methods and, in the capstone case studies of Chapter 10, complex real-world design problems are attacked using all the methods in a unified way.

- **CHALLENGE** *New ideas continue to be introduced into control.*

Control is an active field of research and hence there is a steady influx of new concepts, ideas, and techniques. In time, some of these elements develop to the point where they join the list of things every control engineer

must know. This text is devoted to supporting students equally in their need to grasp both traditional and more modern topics.

In each of our editions we have tried to give equal importance to root locus, frequency response, and state-variable methods for design. In this edition we continue to emphasize solid mastery of the underlying techniques, coupled with computer-based methods for detailed calculation. We also provide an early introduction to data sampling and discrete controllers in recognition of the major role played by digital controllers in our field. While this material can be skipped to save time without harm to the flow of the text, we feel that it is very important for students to understand that computer control is widely used and that the most basic techniques of computer control are easily mastered.

- **CHALLENGE** *Students need to manage a great deal of information.*

The vast array of systems to which feedback control is applied and the growing variety of techniques available for the solution of control problems means that today's student of feedback control must learn many new ideas. How do students keep their perspective as they plow through lengthy and complex textual passages? How do they identify highlights and draw appropriate conclusions? How do they review for exams? Helping students with these tasks was a criterion for the Fourth, Fifth, and Sixth Editions and continues to be addressed in this Seventh Edition. We outline these features below.

FEATURE

1. *Chapter openers* offer perspective and overview. They place the specific chapter topic in the context of the discipline as a whole and they briefly overview the chapter sections.
2. *Margin notes* help students scan for chapter highlights. They point to important definitions, equations, and concepts.
3. *Shaded highlights* identify key concepts within the running text. They also function to summarize important design procedures.
4. *Bulleted chapter summaries* help with student review and prioritization. These summaries briefly reiterate the key concepts and conclusions of the chapter.
5. *Synopsis of design aids*. Relationships used in design and throughout the book are collected inside the back cover for easy reference.
6. *The color blue* is used (1) to highlight useful pedagogical features, (2) to highlight components under particular scrutiny within block diagrams, (3) to distinguish curves on graphs, and (4) to lend a more realistic look to figures of physical systems.
7. *Review questions* at the end of each chapter with solutions in the back to guide the student in self-study
8. *Historical perspectives* at the end of each chapter provide some background and color on how or why the material in that particular chapter evolved.

- **CHALLENGE** *Students of feedback control come from a wide range of disciplines.*

Feedback control is an interdisciplinary field in that control is applied to systems in every conceivable area of engineering. Consequently, some schools have separate introductory courses for control within the standard disciplines and some, like Stanford, have a single set of courses taken by students from many disciplines. However, to restrict the examples to one field is to miss much of the range and power of feedback but to cover the whole range of applications is overwhelming. In this book we develop the interdisciplinary nature of the field and provide review material for several of the most common technologies so that students from many disciplines will be comfortable with the presentation. For Electrical Engineering students who typically have a good background in transform analysis, we include in Chapter 2 an introduction to writing equations of motion for mechanical mechanisms. For mechanical engineers, we include in Chapter 3 a review of the Laplace transform and dynamic response as needed in control. In addition, we introduce other technologies briefly and, from time to time, we present the equations of motion of a physical system without derivation but with enough physical description to be understood from a response point of view. Examples of some of the physical systems represented in the text include the read–write head for a computer disk drive, a satellite tracking system, the fuel–air ratio in an automobile engine, and an airplane automatic pilot system.

Outline of the Book

The contents of the printed book are organized into ten chapters and three appendices. Optional sections of advanced or enrichment material marked with a triangle (Δ) are included at the end of some chapters. Examples and problems based on this material are also marked with a triangle (Δ). There are also four full appendices on the website plus numerous appendices that supplement the material in most of the chapters. The appendices in the printed book include Laplace transform tables, answers to the end-of-chapter review questions, and a list of Matlab commands. The appendices on the website include a review of complex variables, a review of matrix theory, some important results related to state-space design, and optional material supporting or extending several of the chapters.

In Chapter 1, the essential ideas of feedback and some of the key design issues are introduced. This chapter also contains a brief history of control, from the ancient beginnings of process control to flight control and electronic feedback amplifiers. It is hoped that this brief history will give a context for the field, introduce some of the key figures who contributed to its development, and provide motivation to the student for the studies to come.

Chapter 2 is a short presentation of dynamic modeling and includes mechanical, electrical, electromechanical, fluid, and thermodynamic

devices. This material can be omitted, used as the basis of review homework to smooth out the usual nonuniform preparation of students, or covered in-depth depending on the needs of the students.

Chapter 3 covers dynamic response as used in control. Again, much of this material may have been covered previously, especially by electrical engineering students. For many students, the correlation between pole locations and transient response and the effects of extra zeros and poles on dynamic response represent new material. Stability of dynamic systems is also introduced in this chapter. This material needs to be covered carefully.

Chapter 4 presents the basic equations and transfer functions of feedback along with the definitions of the sensitivity function. With these tools, open-loop and closed-loop control are compared with respect to disturbance rejection, tracking accuracy, and sensitivity to model errors. Classification of systems according to their ability to track polynomial reference signals or to reject polynomial disturbances is described with the concept of system type. Finally, the classical proportional, integral, and derivative (PID) control structure is introduced and the influence of the controller parameters on a system's characteristic equation is explored along with PID tuning methods.

Following the overview of feedback in Chapter 4, the core of the book presents the design methods based on root locus, frequency response, and state-variable feedback in Chapters 5, 6, and 7, respectively.

Chapter 8 develops the tools needed to design feedback control for implementation in a digital computer. However, for a complete treatment of feedback control using digital computers, the reader is referred to the companion text, *Digital Control of Dynamic Systems*, by Franklin, Powell, and Workman; Ellis-Kagle Press, 1998.

In Chapter 9 the nonlinear material includes techniques for the linearization of equations of motion, analysis of zero memory nonlinearity as a variable gain, frequency response as a describing function, the phase plane, Lyapunov stability theory, and the circle stability criterion.

In Chapter 10 the three primary approaches are integrated in several case studies and a framework for design is described that includes a touch of the real-world context of practical control design.

Course Configurations

The material in this text can be covered flexibly. Most first-course students in controls will have some dynamics and Laplace transforms. Therefore, Chapter 2 and most of Chapter 3 would be a review for those students. In a ten-week quarter, it is possible to review Chapter 3, and cover all of Chapters 1, 4, 5, and 6. Most optional sections should be omitted. In the second quarter, Chapters 7 and 9 can be covered comfortably including the optional sections. Alternatively, some optional sections could be omitted and selected portions of Chapter 8 included. A semester course should comfortably accommodate Chapters 1–7, including the review materials of Chapters 2 and 3, if needed. If time remains after this core coverage, some

introduction of digital control from Chapter 8, selected nonlinear issues from Chapter 9, and some of the case studies from Chapter 10 may be added.

The entire book can also be used for a three-quarter sequence of courses consisting of modeling and dynamic response (Chapters 2 and 3), classical control (Chapters 4–6), and modern control (Chapters 7–10).

Two basic 10-week courses are offered at Stanford and are taken by seniors and first-year graduate students who have not had a course in control, mostly in the departments of Aeronautics and Astronautics, Mechanical Engineering, and Electrical Engineering. The first course reviews Chapters 2 and 3 and covers Chapters 4–6. The more advanced course is intended for graduate students and reviews Chapters 4–6 and covers Chapters 7–10. This sequence complements a graduate course in linear systems and is the prerequisite to courses in digital control, nonlinear control, optimal control, flight control, and smart product design. Some of the subsequent courses include extensive laboratory experiments. Prerequisites for the course sequence include dynamics or circuit analysis and Laplace transforms.

Prerequisites to This Feedback Control Course

This book is for a first course at the senior level for all engineering majors. For the core topics in Chapters 4–7, prerequisite understanding of modeling and dynamic response is necessary. Many students will come into the course with sufficient background in those concepts from previous courses in physics, circuits, and dynamic response. For those needing review, Chapters 2 and 3 should fill in the gaps.

An elementary understanding of matrix algebra is necessary to understand the state-space material. While all students will have much of this in prerequisite math courses, a review of the basic relations is given in Appendix WB and a brief treatment of particular material needed in control is given at the start of Chapter 7. The emphasis is on the relations between linear dynamic systems and linear algebra.

Supplements

The website mentioned above includes the dot-m and dot-mdl files used to generate all the Matlab figures in the book, and these may be copied and distributed to the students as desired. The website also contains some more advanced material and appendices which are outlined in the Table of Contents. A Solutions Manual with complete solutions to all homework problems is available to instructors only.

Acknowledgments

Finally, we wish to acknowledge our great debt to all those who have contributed to the development of feedback control into the exciting field it is today and specifically to the considerable help and education we have

received from our students and our colleagues. In particular, we have benefited in this effort by many discussions with the following who taught introductory control at Stanford: A. E. Bryson, Jr., R. H. Cannon, Jr., D. B. DeBra, S. Rock, S. Boyd, C. Tomlin, P. Enge, A. Okamura, and C. Gerdes. Other colleagues who have helped us include D. Fraser, N. C. Emami, B. Silver, M. Dorfman, D. Brennan, K. Rudie, L. Pao, F. Khorrami, K. Lorell, M. Tischler, D. de Roover, and P. D. Mathur.

Special thanks go to the many students who have provided almost all the solutions to the problems in the book.

Pearson Education wishes to thank M. J. Anand and V. C. Chandrashekara of PES College of Engineering, and Anil Kumar for reviewing the Global Edition.

Tribute to Gene Franklin

It is with great personal sadness that we report the passing of Prof. Gene Franklin on August 9, 2012. He participated in the initial planning for this edition and contributed to the rewriting of some of the material in Chapter 3. Gene was a mentor, teacher, advisor, and good friend to us both. We are especially proud to have been his friend. We have had a multitude of meetings as we collaborated on the writing of this textbook's editions over 28 years, and every single one of those meetings has been friendly and enjoyable. We each have expressed different viewpoints over the years on how to present various topics, but we were always able to encompass the views into the book in a friendly and collaborative manner. We learned control along with humor from Gene in grad school classes, and we benefitted from his mentoring: in one case as a new Assistant Professor, and in the other as a Ph.D. advisee. Collectively, we have collaborated on research, created new courses and laboratories, and written two textbooks over a period of 40 years. Gene always had a smile with a twinkle in his eye and was a pleasure to work with. We have lost a dear friend and colleague. Gene was a true gentleman.

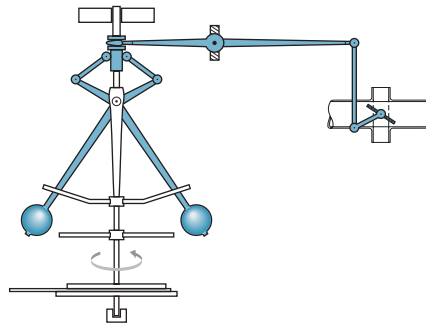
J.D.P.

A.E.-N.

Stanford, California

1

An Overview and Brief History of Feedback Control



A Perspective on Feedback Control

Feedback control of dynamic systems is a very old concept with many characteristics that have evolved over time. The central idea is that a dynamic system's output can be measured and fed back to a controller of some kind and used to affect the system. There are several variations on this theme.

A system that involves a person controlling a machine, as in driving an automobile, is called **manual control**. A system that involves machines only, as when room temperature can be set by a thermostat, is called **automatic control**. Systems designed to hold an output steady against unknown disturbances are called **regulators**, while systems designed to track a reference signal are called **tracking** or **servo** systems. Control systems are also classified according to the information used to compute the controlling action. If the controller does *not* use a measure of the system output being controlled in computing the control action to take, the system is called **open-loop control**. If the controlled output signal *is* measured and fed back for use in the control computation, the system is called **closed-loop** or **feedback control**. There are many other important properties of control systems in addition to these most basic characteristics. For example, we will mainly consider feedback of current measurements

as opposed to predictions of the future; however, a very familiar example illustrates the limitation imposed by that assumption. When driving a car, the use of simple feedback corresponds to driving in a thick fog where one can *only see the road immediately at the front of the car* and is unable to see the future required position! Looking at the road ahead is a form of predictive control and this information, which has obvious advantages, would always be used where it is available; but in most automatic control situations studied in this book, observation of the future track or disturbance is not possible. In any case, the control designer should study the process to see if any information could anticipate either a track to be followed or a disturbance to be rejected. If such a possibility is feasible, the control designer should use it to **feedforward** an early warning to the control system. An example of this is in the control of steam pressure in the boiler of an electric power generation plant. The electricity demand cycle over a day is well known; therefore, when it is known that there will soon be an increased need for electrical power, that information can be fed forward to the boiler controller in anticipation of a soon-to-be-demanded increase in steam flow.

The applications of feedback control have never been more exciting than they are today. Automatic landing and collision avoidance systems are now being used and the use of satellite navigation in future designs promises a revolution in our ability to navigate in an ever more crowded airspace. In the magnetic data storage devices for computers known as hard disks, control of the read/write head assembly is often designed to have tracking errors on the order of microns and to move at speeds of a fraction of a millisecond. Control is essential to the operation of systems from cell phones to jumbo jets and from washing machines to oil refineries as large as a small city. Applications of control to driverless cars and surgical robotic systems are emerging. The list goes on and on. In fact, many engineers refer to control as a *hidden technology* because of its essential importance to so many devices and systems while being mainly out of sight. The future will no doubt see engineers create even more imaginative applications of feedback control.

Chapter Overview

In this chapter we begin our exploration of feedback control using a simple familiar example: a household furnace controlled by a thermostat. The generic components of a control system are identified within the context of this example. In another example in Section 1.2—an automobile cruise control—we develop the elementary static equations and assign numerical values to elements of the system model in order to compare the performance of open-loop control to that of feedback control when dynamics are ignored. Section 1.3 then introduces the key elements in control system design. In order to

provide a context for our studies and to give you a glimpse of how the field has evolved, Section 1.4 provides a brief history of control theory and design. In addition, later chapters have brief sections of additional historical notes on the topics covered there. Finally, Section 1.5 provides a brief overview of the contents and organization of the entire book.

1.1 A Simple Feedback System

In feedback systems the variable being controlled—such as temperature or speed—is measured by a sensor and the measured information is fed back to the controller to influence the controlled variable. The principle is readily illustrated by a very common system, the household furnace controlled by a thermostat. The components of this system and their interconnections are shown in Fig. 1.1. Such a picture identifies the major parts of the system and shows the directions of information flow from one component to another.

We can easily analyze the operation of this system qualitatively from the graph. Suppose both the temperature in the room where the thermostat is located and the outside temperature are significantly below the reference temperature (also called the set point) when power is applied. The thermostat will be *on* and the control logic will open the furnace gas valve and light the fire box. This will cause heat Q_{in} to be supplied to the house at a rate that will be significantly larger than the heat loss Q_{out} . As a result, the room temperature will rise until it exceeds the thermostat reference setting by a small amount. At this time the furnace will be turned off and the room temperature will start to fall toward the outside value. When it falls a small amount below the set point, the thermostat will come on again and the cycle will repeat. Typical plots of room temperature along with the furnace cycles of on and off are shown in Fig. 1.1. The outside temperature remains at 50°F and the thermostat is initially set at 55°F. At 6 a.m., the thermostat is stepped to 65°F and the furnace brings it to that level and cycles the temperature around that figure thereafter.¹ Notice that the house is well insulated, so that the fall of temperature with the furnace off is significantly slower than the rise with the furnace on. From this example, we can identify the generic components of the elementary feedback control system as shown in Fig. 1.2.

The central component of this feedback system is the **process** whose output is to be controlled. In our example the process would be the house whose output is the room temperature and the **disturbance** to the process is the flow of heat from the house, Q_{out} , due to conduction through the walls and roof to the lower outside temperature. (The outward flow of heat also depends on other factors such as wind, open doors, and so on.) The design of the process can obviously have a major impact on the effectiveness of the controls. The temperature of a well-insulated house with thermopane windows is clearly easier to control than otherwise. Similarly, the design of

¹Notice that the furnace had come on a few minutes before 6 a.m. on its regular nighttime schedule.

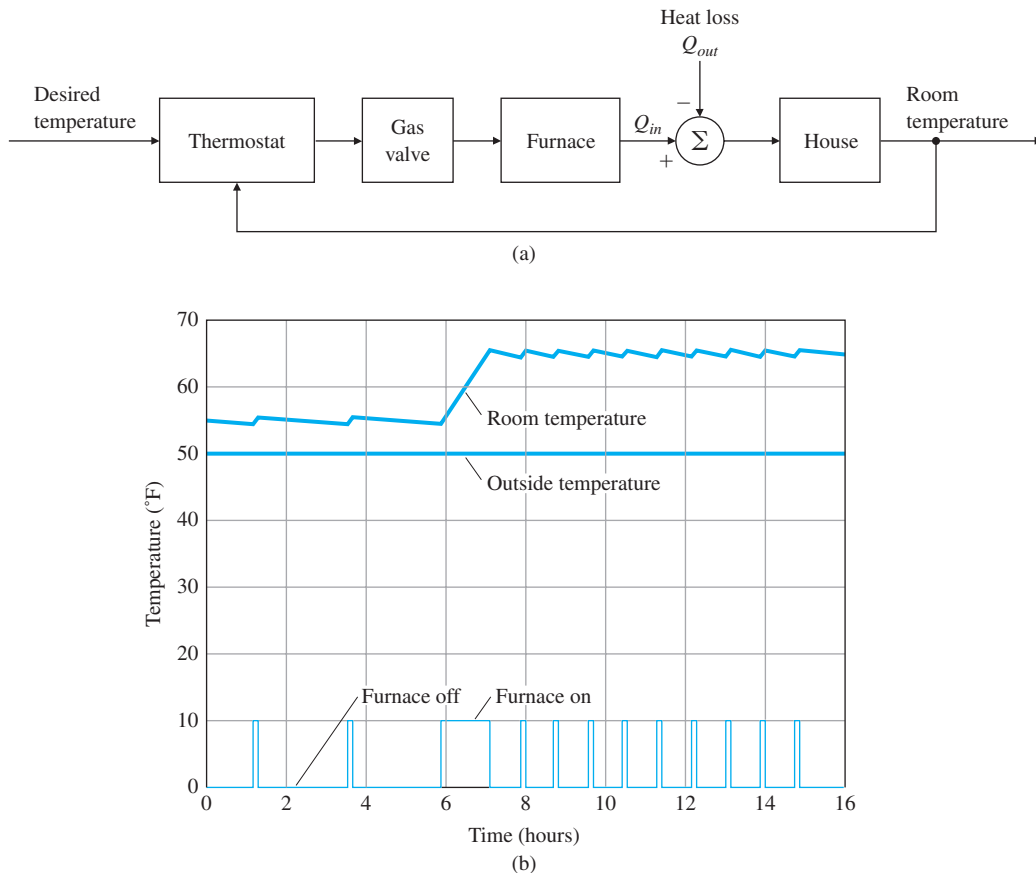


Figure 1.1

Feedback control: (a) component block diagram of a room temperature control system; (b) plot of room temperature and furnace action

aircraft with control in mind makes a world of difference to the final performance. In every case, the earlier the issues of control are introduced into the process design, the better. The **actuator** is the device that can influence the controlled variable of the process and in our case, the actuator is a gas furnace. Actually, the furnace usually has a pilot light or striking mechanism, a gas valve, and a blower fan, which turns on or off depending on the air temperature in the furnace. These details illustrate the fact that many feedback systems contain components that themselves form other feedback systems.² The central issue with the actuator is its ability to move the process output with adequate speed and range. The furnace must produce more heat than the house loses on the worst day and must distribute it quickly if the house temperature is to be kept in a narrow range. Power, speed, and reliability

²Jonathan Swift (1733) said it this way: “So, Naturalists observe, a flea Hath smaller fleas that on him prey; And these have smaller still to bite ‘em; And so proceed, *ad infinitum*.”

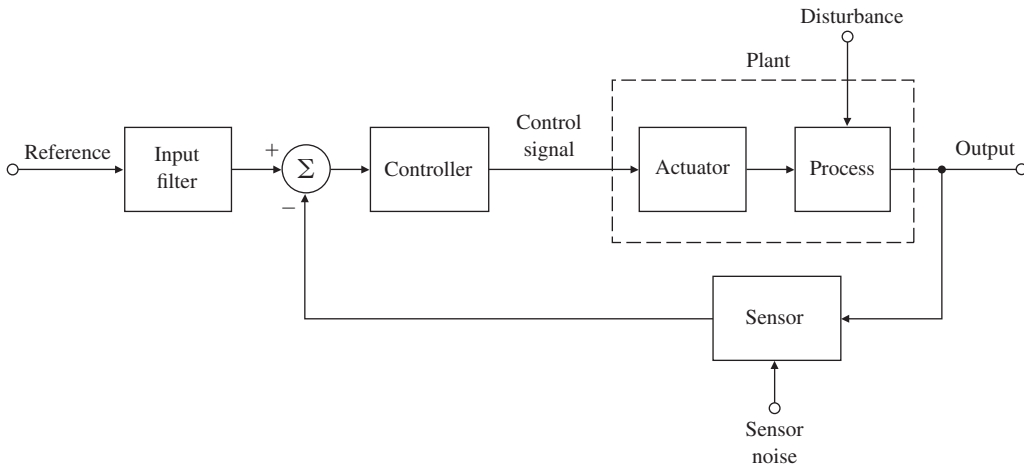


Figure 1.2
Component block diagram of an elementary feedback control

are usually more important than accuracy. Generally, the process and the actuator are intimately connected and the control design centers on finding a suitable input or control signal to send to the actuator. The combination of process and actuator is called the **plant** and the component that actually computes the desired control signal is the **controller**. Because of the flexibility of electrical signal processing, the controller typically works on electrical signals although the use of pneumatic controllers based on compressed air has a long and important place in process control. With the development of digital technology, cost-effectiveness and flexibility have led to the use of digital signal processors as the controller in an increasing number of cases. The component labeled **thermostat** in Fig. 1.1 measures the room temperature and is called the **sensor** in Fig. 1.2, a device whose output inevitably contains sensor noise. Sensor selection and placement are very important in control design, for it is sometimes not possible for the true controlled variable and the sensed variable to be the same. For example, although we may really wish to control the house temperature as a whole, the thermostat is in one particular room, which may or may not be at the same temperature as the rest of the house. For instance, if the thermostat is set to 68°F but is placed in the living room near a roaring fireplace, a person working in the study could still feel uncomfortably cold.^{3,4} As we will see, in addition to placement,

³In the renovations of the kitchen in the house of one of the authors, the new ovens were placed against the wall where the thermostat was mounted on the other side. Now when dinner is baked in the kitchen on a cold day, the author freezes in his study unless the thermostat is reset.

⁴The story is told of the new employee at the nitroglycerin factory who was to control the temperature of a critical part of the process manually. He was told to “keep that reading below 300°.” On a routine inspection tour, the supervisor realized that the batch was dangerously hot and found the worker holding the thermometer under cold water tap to bring it down to 300°. They got out just before the explosion. Moral: sometimes automatic control is better than manual.

important properties of a sensor are the accuracy of the measurements as well as low noise, reliability, and linearity. The sensor will typically convert the physical variable into an electrical signal for use by the controller. Our general system also includes an **input filter** whose role is to convert the reference signal to electrical form for later manipulation by the controller. In some cases the input filter can modify the reference command input in ways that improve the system response. Finally, there is a **controller** to compute the difference between the reference signal and the sensor output to give the controller a measure of the system error. The thermostat on the wall included the sensor, input filter, and the controller. A few decades ago, the user simply set the thermostat manually to achieve the desired room temperature at the thermostat location. Over the last few decades, the addition of a small computer in the thermostat has enabled storing the desired temperature over the day and week and more recently, thermostats have gained the ability to learn what the desired temperature should be and to base that value, in part, on whether anybody will be home soon!⁵

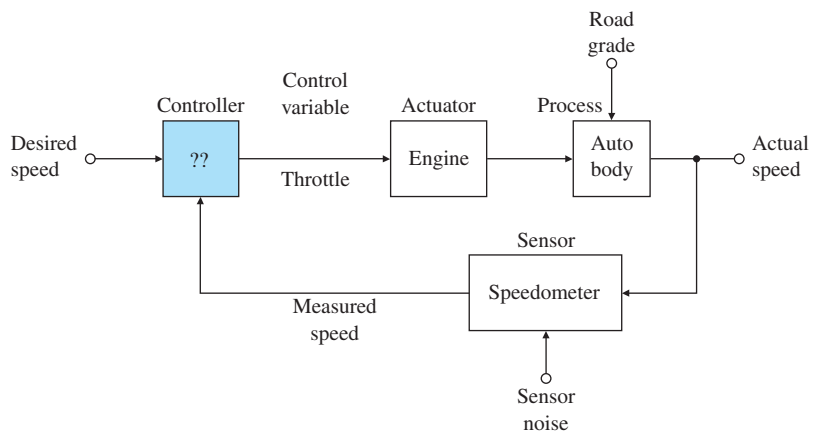
This text will present methods for analyzing feedback control systems and will describe the most important design techniques engineers can use in applying feedback to solve control problems. We will also study the specific advantages of feedback that compensate for the additional complexity it demands.

1.2 A First Analysis of Feedback

The value of feedback can be readily demonstrated by quantitative analysis of a simplified model of a familiar system, the cruise control of an automobile (Fig. 1.3). To study this situation analytically, we need a mathematical **model** of our system in the form of a set of quantitative relationships among the

Figure 1.3

Component block diagram of automobile cruise control



⁵The sensor has a motion detector to determine whether anybody is home and the user can command the unit from afar via the Internet. One example is by Nest, who was recently purchased by Google. See: www.nest.com

variables. For this example, we ignore the dynamic response of the car and consider only the steady behavior. (Dynamics will, of course, play a major role in later chapters.) Furthermore, we assume that for the range of speeds to be used by the system, we can approximate the relations as linear. After measuring the speed of the vehicle on a level road at 65 mph, we find that a 1° change in the throttle angle (our control variable, u) causes a 10 mph change in speed (the output variable, y), hence the value 10 in the box between u and y in Fig. 1.4, which is a **block diagram** of the plant. Generally, the block diagram shows the mathematical relationships of a system in graphical form. From observations while driving up and down hills, it is found that when the grade changes by 1%, we measure a speed change of 5 mph, hence the value 0.5 in the upper box in Fig. 1.4, which reflects that a 1% grade change has half the effect of a 1° change in the throttle angle. The speedometer is found to be accurate to a fraction of 1 mph and will be considered exact. In the block diagram the connecting lines carry signals and a block is like an ideal amplifier which multiplies the signal at its input by the value marked in the block to give the output signal. To sum two or more signals, we show lines for the signals coming into a summer, a circle with the summation sign Σ inside. An algebraic sign (plus or minus) beside each arrow head indicates whether the input adds to or subtracts from the total output of the summer. For this analysis, we wish to compare the effects of a 1% grade on the output speed when the reference speed is set for 65 with and without feedback to the controller.

In the first case, shown in Fig. 1.5, the controller does not use the speedometer reading but sets $u = r/10$, where r is the reference speed, which is, 65 mph. This is an example of an **open-loop control system**. The

Open-loop control

Figure 1.4
Block diagram of the
cruise control plant

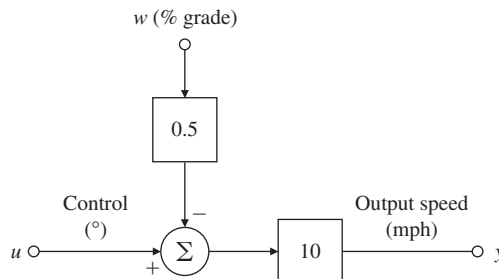
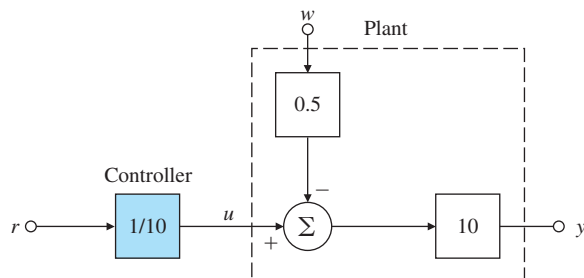


Figure 1.5
Open-loop cruise
control



term *open loop* refers to the fact that there is no closed path or loop around which the signals go in the block diagram; that is, the control variable u is independent of the output variable, y . In our simple example, the open-loop output speed, y_{ol} , is given by the equations

$$\begin{aligned} y_{ol} &= 10(u - 0.5w) \\ &= 10\left(\frac{r}{10} - 0.5w\right) \\ &= r - 5w. \end{aligned}$$

The error in output speed is

$$e_{ol} = r - y_{ol} \quad (1.1)$$

$$= 5w, \quad (1.2)$$

and the percent error is

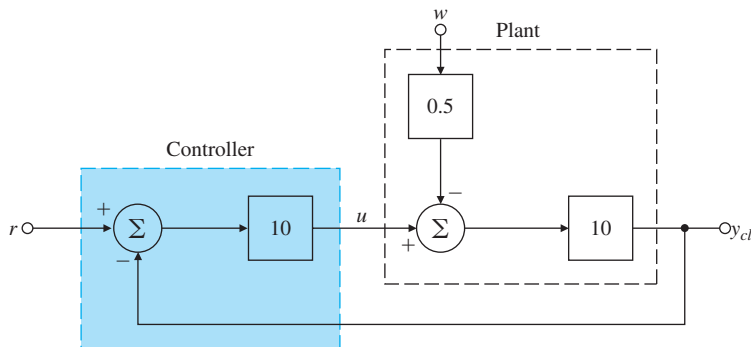
$$\% \text{ error} = 500 \frac{w}{r}. \quad (1.3)$$

If $r = 65$ and the road is level, then $w = 0$ and the speed will be 65 with no error. However, if $w = 1$ corresponding to a 1% grade, then the speed will be 60 and we have a 5-mph error, which is a 7.69% error in the speed. For a grade of 2%, the speed error would be 10 mph, which is an error of 15.38%, and so on. The example shows that there would be no error when $w = 0$, but this result depends on the controller gain being the exact inverse of the plant gain of 10. In practice, the plant gain is subject to change and if it does, errors are introduced by this means also. If there is an error in the plant gain in open-loop control, the percent speed error would be the same as the percent plant-gain error.

The block diagram of a feedback scheme is shown in Fig. 1.6, where the controller gain has been set to 10. In this simple example, we have assumed that we have an ideal sensor providing a measurement of y_{cl} . In this case the equations are

Figure 1.6

Closed-loop cruise control



$$y_{cl} = 10u - 5w,$$

$$u = 10(r - y_{cl}).$$

Combining them yields

$$y_{cl} = 100r - 100y_{cl} - 5w,$$

$$101y_{cl} = 100r - 5w,$$

$$y_{cl} = \frac{100}{101}r - \frac{5}{101}w,$$

$$e_{cl} = \frac{r}{101} + \frac{5w}{101}.$$

Thus the feedback has reduced the sensitivity of the speed error to the grade by a factor of 101 when compared with the open-loop system. Note, however, that there is now a small speed error on level ground because even when $w = 0$,

$$y_{cl} = \frac{100}{101}r = 0.99r \text{ mph.}$$

This error will be small as long as the loop gain (product of plant and controller gains) is large.⁶ If we again consider a reference speed of 65 mph and compare speeds with a 1% grade, the percent error in the output speed is

$$\% \text{ error} = 100 \frac{\frac{65 \times 100}{101} - \left(\frac{65 \times 100}{101} - \frac{5}{101} \right)}{\frac{65 \times 100}{101}} \quad (1.4)$$

$$= 100 \frac{5 \times 101}{101 \times 65 \times 100} \quad (1.5)$$

$$= 0.0769\%. \quad (1.6)$$

The reduction of the speed sensitivity to grade disturbances and plant gain in our example is due to the loop gain of 100 in the feedback case. Unfortunately, there are limits to how high this gain can be made; when dynamics are introduced, the feedback can make the response worse than before, or even cause the system to become unstable. The dilemma is illustrated by another familiar situation where it is easy to change a feedback gain. If one tries to raise the gain of a public-address amplifier too much, the sound system will squeal in a most unpleasant way. This is a situation where the gain in the feedback loop—from the speakers to the microphone through the amplifier back to the speakers—is too much. The issue of how

The design
trade-off

⁶In case the error is too large, it is common practice to *reset* the reference, in this case to $\frac{101}{100}r$, so the output reaches the true desired value.

to get the gain as large as possible to reduce the errors without making the system become unstable and squeal is what much of feedback control design is all about.

1.3 Feedback System Fundamentals

To achieve good control there are typical goals:

- **Stability:** The system must be stable at all times. This is an absolute requirement.
- **Tracking:** The system output must track the command reference signal as closely as possible.
- **Disturbance rejection:** The system output must be as insensitive as possible to disturbance inputs.
- **Robustness:** The aforementioned goals must be met even if the model used in the design is not completely accurate or if the dynamics of the physical system change over time.

The requirement of **stability** is basic and instability may have two causes. In the first place, the system being controlled may be unstable. This is illustrated by the Segway vehicle, which will simply fall over if the control is turned off. A second cause of instability may be the addition of feedback! Such an instability is called a “vicious circle,” where the feedback signal that is circled back makes the situation worse rather than better. Stability will be discussed in much more detail in Chapters 3 and 4.

There are many examples of the requirement of having the system’s output track a command signal. For example, driving a car so that the vehicle stays in its lane is **command tracking**. Today, this is done by the driver; however, there are schemes now under development where the car’s “auto-driver” will carry out this task using feedback control while the driver does other things, for example, surfing the Internet. Similarly, flying an airplane on the approach to landing requires that a glide path be accurately tracked by the pilot or an autopilot. It is routine for today’s aircraft autopilots to carry this out including the flare to the actual touchdown. The autopilot accepts inputs from the Instrument Landing System (ILS) that provides an electronic signal showing the desired landing trajectory, then commands the aircraft control surfaces so that it follows the desired trajectory as closely as possible.

Disturbance rejection is one of the very oldest applications of feedback control. In this case, the “command” is simply a constant set point to which the output is to be held as the environment changes. A very common example of this is the room thermostat whose job it is to hold the room temperature close to the set point as outside temperature and wind change, and as doors and windows are opened and closed.

Finally, to design a controller for a dynamic system, it is necessary to have a **mathematical model** of the dynamic response of the system being controlled in all but the simplest cases. Unfortunately, almost all physical systems are very complex and often nonlinear. As a result, the design will